Hypercomplex Seminar 2023

July 9–14

Organisers

Institute of Mathematics,
Polish Academy of Science

Faculty of Natural and Technical Sciences,
The John Paul II Catholic University of Lublin

Institute of Mathematics,
The University College of Applied Sciences in Chelm

Department of Microelectronics and Computer Science,
Lodz University of Technology

Faculty of Electrical, Electronic, Computer and Control Engineering,
Lodz University of Technology

Faculty of Computer Science and Telecommunications,
Cracow University of Technology

Honorary organisers

Department of Solid State Physics, Faculty of Physics and Applied Informatics,
University of Lodz

Łódź Scientific Society

Partners

Quantum Hardware Systems

Program and abstracts

(2023-07-14)

July 9, 2023

Opening ceremony (17:45-17:55)

Session (Inaugural lectures),
Chairmen: Dariusz Partyka and Mariusz Zubert

18:00-18:50 *Sergiy Plaksa*, (→)
Monogenic functions in infinite-dimensional vector spaces with a commutative multiplication and harmonic vectors

19:00-19:50 *Marek Danielewski*, (→)
Quaternion Quantum Mechanics II: Unraveling the Mysteries of Gravity and the Dirac Equation within the Planck-Kleinert Crystal

20:00-20:50 *Chantal Roth*, (→)
Quaternionic foundations of Quantum Mechanics and spin 1/2 visualization

July 10, 2023

Session A1 (Hypercomplex structures),
Chairman: Sergiy Plaksa

10:00-10:50 *Lino F. Reséndis O.*, (→)
$\mathcal{Q}_p$ spaces for hyperholomorphic functions in the unit ball of $\mathbb{R}^4$

11:00-11:25 *Serhii Gryshchuk*, (→)
Representations of solutions of Lamé system with real coefficients via monogenic functions in the biharmonic algebra

11:30-12:20 *Massimo Vaccaro*, (→)
The full set of invariants characterizing the $Sp(n)$-orbits in the real Grassmannians $G^{\mathbb{R}}(k, 4n)$

Session A2 (Complex and real analysis of one variable),
Chairman: Lino F. Reséndis Ocampo

13:00-13:25 *Mariia Stefanchuk*, (→)
On the exponential asymptotic of solutions of the nonlinear Cauchy-Riemann-Beltrami type equation

July 11, 2023

Session A3 (Holomorphic functions of one variable),
Chairman: Serhii Gryshchuk

10:00-10:25 *Iryna Denega*, (→)
Evolutionary type inequalities for products of inner radii

10:30-10:55 *Maciej Parol*, (→)
The Koebe radius for certain class of polynomials

11:00-11:25 *Szymon Ignaciuk*, (→)
Physical interpretation of the membership to Kaplan classes of certain functions

11:30-11:55 *Jacek Dziok*, (→)
On multivalent prestarlike functions

Session A4 (Complex and real analysis of several variables),
Chairman: Massimo Vaccaro

12:30-12:55 *Valery Volchkov*, *Vitaly Volchkov*, (→)
Continuous mean periodic extension of functions from a segment

13:00-13:25 *Anna Kimaczyńska*, (→)
The grad div operator

Session A5 (Poster session),
Chairman: Massimo Vaccaro

13:30-14:00 *Olga Rovenska*, (→)
Approximation of classes of Poisson integrals by Fejér means

13:30-14:00 *Natalia Zorii*, (→)
On the role of the point at infinity in Deny's principle of positivity of mass for Riesz potentials

July 12, 2023

Session B1 (Physics: Ontology of Quantum Mechanics),
Chairman: Krzysztof Pomorski

10:00-10:50 *Jarosław Duda*, (→)
Topological charges with electromagnetic + gravitomagnetic interactions

11:00-11:50 *Richard Kerner*, (→)
Non-linear electrodynamics derived from the Kaluza-Klein 5-dimensional gravity

12:30-13:20 *Manfried Faber*, (→)
Running coupling from a classical soliton model

13:30-13:55 *Łukasz Stępień*, (→)
On some exact solutions of certain nonlinear partial differential equations

14:00-14:50 *Jarosław Duda*, (→)
Maximal Entropy Random Walk to understand Born rule and Bell violation

Session A6 & B (Advanced mathematical methods in physics),
Chairman: Jarosław Duda

15:00-15:50 *Osamo Suzuki*, (→)
Binary and ternary structures in evolution theory (VI) An entropic description of the evolution of the universe

16:00-16:25 *Manfried Faber*, (→)
Running coupling from a classical soliton model (continuation)

July 13, 2023

Session C1 & B (Quantum Computer Science),
Chairman: Chantal Roth

10:00-10:50 *K. Gnatenko*, (→)
Geometric measure of entanglement of variational quantum states and its quantifying on a quantum computer

11:00-11:50 *K. Pomorski*, *D. Kotula*, (→)
Classical and Quantum Conway Game of Life

Session C2 & B (Classical/Quantum Electronics and Machine Learning),
Chairman: Khrystyna Gnatenko

12:30-12:55 *M. Kowalik*, (→)
The application of unsupervised learning to the AC susceptibility measurements of High-Temperature Superconductors

13:00-13:25 *M. Zubert*, *Z. Kulesza*, *M. Jankowski*, *A. Napieralski*, (→)
Application of Scattering Parameters to The DPL Time-Lag Parameter Estimation using MEMS Structures

Session D1 & B (Classical Electronics),
Chairman: Mariusz Zubert

13:30-13:55 *M. Złotkowski*, (→)
Local systems management of different energy sources

Discussion panel (14:00-14:30)
Chairmen: Sergiy Plaksa, Manfried Faber, Chantal Roth

July 14, 2023

Session D2 & B (Quantum Electronics),
Chairman: Krzysztof Pomorski

10:00-10:50 *M. Belogolovskii*, (→)
Two new issues in superconducting electronics: transparent samples and SQUID-like Josephson junctions

11:00-11:50 *J. Sosnowski*, (→)
Influence of quantized vortex dynamics on superconducting electronics

12:00-12:25 *V. Vachtsevanos*, (→)
Emergence of topological defects in finite 2-D lattices during second phase order transitions

12:30-12:55 *Oleh Yermakov*, (→)
Discovery of polarization degree of freedom for localized light

Session A7 (Geometrical methods in physics),
Chairman: Andrzej Michalski

13:30-13:55 *A. Paszkiewicz*, (→)
Does the convex Peano curve exist and what can we say about operators?

14:00-14:25 *Armen Grigoryan*, (→)
Landau and Bloch theorems for planar harmonic functions

14:30-14:55 *Małgorzata Nowak-Kępczyk*, (→)
Iterative construction of discrete Laplacians on 2D square and hexagonal lattices using BTBB and other sequences

Session C3 & B & D (Computer Science in Quantum Electronics),
Chairman: Mariusz Zubert

15:30-15:55 *K. Pomorski*, (→)
Universal modeling of electrostatic semiconductor quantum gates of any topology interfaced to Josephson junction quantum circuit

16:00-16:50 *B. Stojewski*, *K. Pomorski*, (→)
Hybrid Schrödinger-Ginzburg-Landau (Sch-GL) approach in study of superconducting integrated structures

17:00-17:25 *Ł. Pluszyński*, *K. Pomorski*, (→)
Towards construction of analog solver of Schrödinger equation based on Kron's second model

17:30-17:55 *Ł. Pluszyński*, (→)
Simulation and experiments with superconducting single photon detectors

Discussion panel and Closing ceremony (18:00-18:30)

Publication of conference manuscripts is possible in one among three journals:

- **Technical Transactions** (mathematical and non-mathematical technical sciences);
- **Elsevier Mathematics and Computers in Simulations** (mathematical structures and computer sciences);
- **MDPI Quantum Reports** (Special issue on semiconductor and quantum devices; publication is free for Ukrainian citizens).

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July 15, 2023

Jarosław Duda and Krzysztof Pomorski
recommend topics on Ontology of Quantum Mechanics
(QM Foundations & Nature of Time seminar)

16:00-17:00 *Sergey Rashkovskiy (RAS)*, `http://th.if.uj.edu.pl/~dudaj/QMFNoT`
Quantum Mechanics: Strange Particle Theory or Classical Field Theory?

Mikhail Belogolovskii
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Two new issues in superconducting electronics: transparent samples and SQUID-like Josephson junctions

Abstract. Nowadays, superconducting circuits are among the most prominent platforms for quantum computing, communications, and sensing. Transparent superconductors are of particular interest, since they allow direct interfacing between stationary superconducting and flying photonic qubits. The latter are necessary for the transmission of quantum information because their quantum states are more resistant to decoherence. I present a methodological approach for the development of superconductors with acceptable transparency and usefully high critical temperature, followed by discussion of representative characteristics of a number of already known oxides. In particular, I examine how the complex interplay of free carrier absorption and interband transitions can lead to the coexistence of comparatively high carrier density at the Fermi level and the needed transparency for visible light. To date, noticeable successes have been achieved using the first approach and, as a good example of its implementation, we overview detailed results obtained for electrochemically doped indium-tin-oxide compounds, which is a reference material for testing our ideas.

Next, I present our results on synthetic binary superlattices formed by nanometer-thick layers of normal and ferromagnetic metals, quantum transport through which unexpectedly demonstrates the formation of edge states, which are resistant to disorder and other mobile charge interactions. The idea of the experiments was inspired by a paradigmatic explanation of the plateaus in transversal transport characteristics in a 2D conductor at very low temperatures and strong magnetic fields (the integer quantum Hall effect) that is based on the existence of narrow near-boundary quantum channels of non-interacting electrons created at the Fermi level in strongly disordered electron systems. The edge states are chiral in the sense that they can carry current only in one fixed direction. At the same time, the edge modes are topologically protected and their number cannot vary under continuous transformation of the system. This approach assumes the presence of a disordered and isotropic two-dimensional electron gas. Our aim has been to create a strongly anisotropic 3D system with edge channels concentrated mainly at the hinges of the structure, i.e., a synthetic material with one or two 1D conducting modes. One of the best ways to probe spatial current distributions is Josephson interferometry in a stacked configuration where the hybrid structure under study links two superconducting (S) electrodes. The figure of merit in such experiments is the maximum supercurrent (I_c) versus in-plane magnetic field (H). In our experiments, we applied this technique to study the transport of Cooper pairs through a periodic multilayer formed by ten normal metal (Al) - ferromagnetic metal (Ni or Ni-Fe alloy) bilayers. For the first time, we observed SQUID-like ($I_c - H$)) oscillations instead of conventional Fraunhofer patterns expected for trivial S-weak link-S junctions. Possible applications of such devices are discussed.

Keywords and phrases: Josephson junctions, transparent superconductiong films, ferromagnetic-normal metallic superlattices, Josephson interferometry

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Quaternion Quantum Mechanics II: Unraveling the Mysteries of Gravity and the Dirac Equation within the Planck-Kleinert Crystal

Abstract. We present quaternion representation of quantum mechanics that allows its ontological interpretation. The correspondence between classical and quaternion quantum equations, permits considering the universe (vacuum) as an ideal elastic solid. Elementary particles would have to be standing or soliton-like waves. Tension induced by the compression and twisting of the elastic medium would increase the energy density, consequently generate a gravity forcing and affect the wave speed. Consequently the gravity could be described by an index of refraction.

Theory was created by combining the Cauchy model of the elastic continuum with the Planck-Kleinert crystal hypothesis. The quaternion-imaginary Lagrangian, the quaternion motion equation and the quaternionic oscillator allowed deriving:

- The Schrödinger equation from the functional integral, which identifies the quaternion-imaginary quantum Hamiltonian.
- The 2^{nd} order wave equation system describing both the bosons and the gravity.
- The first order quaternionic wave equation system.
- The family of the second order wave equation systems describing both the particles and the generated quaternionic force-fields.
- The quaternionic continuity equation in an ideal elastic solid.

Keywords and phrases: quaternion
Joint work with: Lucjan Sapa, Chantal Roth

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Evolutionary type inequalities for products of inner radii

Abstract. In this presentation we consider evolutionary type inequalities for the following products

$$r(B_0,0)\prod_{k=1}^{n} r(B_k,a_k),$$

$$r(B_\infty,\infty)\prod_{k=1}^{n} r(B_k,a_k),$$

$$r(B_0,0)\,r(B_\infty,\infty)\prod_{k=1}^{n} r(B_k,a_k),$$

where $n\in\mathbb{N}$, $A_n=\{a_k\}_{k=1}^n$ be an arbitrary fixed system of points of the complex plane, B_0, B_∞ and $\{B_k\}_{k=1}^n$ be an arbitrary system of domains such that $a_0=0\in B_0\subset\overline{\mathbb{C}}$, $\infty\in B_\infty\subset\overline{\mathbb{C}}$, $a_k\in B_k\subset\overline{\mathbb{C}}$, $k=\overline{1,n}$, $B_i\cap B_j=\varnothing$, $i\neq j$, $r(B,a)$ denote an inner radius of the domain B with respect to a point a.

Keywords and phrases: the Green function, an inner radius of the domain, logarithmic capacity

References

[1] A. K. Bakhtin, I. V. Denega, *Generalized M.A. Lavrentiev's inequality*, J. Math. Sci. **262** (2022), 138–153, (https://doi.org/10.1007/s10958-022-05806-y).

[2] I. Denega, *Extremal decomposition of a multidimensional complex space with poles on the boundary of a polydisk*, In: Cerejeiras P., Reissig M., Sabadini I., Toft J. (eds) Current Trends in Analysis, its Applications and Computation. Trends in Mathematics. Birkhäuser, Cham. (2022), 143–151, (https://doi.org/10.1007/978-3-030-87502-2_14).

[3] A. K. Bakhtin, I. V. Denega, *Weakened problem on extremal decomposition of the complex plane*, Matematychni Studii **51** (2019), 35–40, (https://doi.org/10.15330/ms.51.1.35-40).

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Jarosław Duda
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Maximal Entropy Random Walk to understand Born rule and Bell violation

Abstract. Turns out standard random walk/diffusion usually only approximates the (Jaynes) maximal entropy principle - necessary for statistical mechanics models. MERW (maximal entropy random walk) as e.g. random walk along Ising sequence repairs this approximation, this way also disagreements with quantum mechanics - like Anderson localization, Born rule and Bell violation. I will introduce to MERW and discuss suggestions for physics understanding, including potential new possibilities and their applications.
Slides here: https://www.dropbox.com/s/a8yqfabq3gxsjth/Bell%20mini.pdf

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Jarosław Duda
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Topological charges with electromagnetic + gravitomagnetic interactions

Abstract. In liquid crystals they experimentally obtain topological charges with long-range e.g. Coulomb-like interactions. It brings question how far can we take its resemblance with particle physics. I will talk about such looking promising approach based on Landau-de Gennes model, with EM-like Lagrangian interpreting field curvature as dual EM field (Faber's approach). This way Gauss law counts topological charge for its quantization, also thanks to Higgs-like potential regularizing charge to finite energy in agreement with the running coupling effect. Then there appear further particle-like topological defects, like topological vortices with knots resembling baryons (e.g. having proton lighter than neutron), and nuclei knotted against Coulomb repulsion. Extending to 4D field adding boosts, their dynamics turns out governed by second set of Maxwell equations for gravity.
Slides here: https://www.dropbox.com/s/9dl2g9lypzqu5hp/liquid%20crystal%20particles.pdf

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On multivalent prestarlike functions

Let $\mathcal{A}_p$ $(p \in \mathbb{N} := \{1, 2, \ldots\})$ denote the class of functions of the form

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n$$

which are analytic in $\mathcal{U} := \{z \in \mathbb{C} : |z| < 1\}$. A function $f \in \mathcal{A}_p$ is said to be *multivalent starlike of order* α $(0 \leq \alpha < p)$ if

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \alpha \quad (z \in \mathcal{U}).$$

We denote by $\mathcal{S}_p^c(\alpha)$ the class of all functions which are starlike of order α.

Due to Ruscheweyh [1] we introduce the class of multivalent prestarlike functions. We say that a function $f \in \mathcal{A}_p$ belongs to the class $\mathcal{R}_p(\alpha)$ of *multivalent prestarlike functions of order* α if

$$f(z) * \frac{z^p}{(1-z)^{2(p-\alpha)}} \in \mathcal{S}_p^*(\alpha),$$

where "$*$" denotes the Hadamard product (or convolution).

The main object of the talk is to present some properties of multivalent prestarlike functions and their applications.

References

[1] S. Ruscheweyh, *Convolutions in geometric function theory*, Sem. Math. Sup. 83, Les Presses del'Université de Montréal, 1982.

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Running coupling from a classical soliton model

Abstract. Running coupling in field theory was found by the necessity to cancel the infinities which appear in the Coulomb field of point-like charges and experimentally verified in high energy collisions of electron-positron pairs. The model of topologogical particles (MTP) describes electrons as topological solitons of finite size and mass, of a field of rotations of spatial Dreibeins without any singularities. Therefore, there is no need to remove any infinities. The question whether MTP is able to describe the running of the electric charge is investigated in numerical computations of the energy of soliton pairs at varying distance. It is shown that within MTP the running charge of electrons originates naturally in the finite size of electrons.

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Khrystyna Gnatenko
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Geometric measure of entanglement of variational quantum states and its quantifying on a quantum computer

Abstract. The entanglement of variational quantum states is studied with analytical calculations and on quantum computers of IBM and Rigetti. We consider variational quantum states prepared with rotational RY, RZ, and RXX gates. The states have a graph structure and can be considered as quantum graph states constructed with the action of the operator of evolution corresponding to the Ising model. In the particular cases of parameters of RY and RZ gates the states were studied in [1]. The entanglement of the states is obtained analytically for the arbitrary structure of a graph. The dependencies of the entanglement on the parameters of the rotational gates are analyzed. The states are prepared on IBM's quantum computer. We calculate the geometric measure of entanglement of the states with quantum programming. The protocol for the calculations is based on the relation of the geometric measure of entanglement with mean spin obtained in [2].

We also study the entanglement of variational quantum states corresponding to the Generative Adversarial Network states [3, 4]. In the case of one-layer, the states also can be considered as quantum graph states [5]. The entanglement of the states is calculated analytically and with quantum programming on IBM's and Rigetti quantum computers [6, 7]. The results of quantum calculations are in good agreement with the theoretical ones.

Keywords and phrases: quantum computers, geometric measure of entanglement, quantum graph states, Generative Adversarial Network states

References

[1] Kh. P. Gnatenko, V. M. Tkachuk, *Entanglement of graph states of spin system with Ising interaction and its quantifying on IBM's quantum computer*, Phys. Lett A. **396**, 127248, 5 p. (2021).
[2] A. M. Frydryszak, M. I. Samar, V. M. Tkachuk
Quantifying geometric measure of entanglement by mean value of spin and spin correlations with application to physical systems Eur. Phys. J. D **71**, 233 [8 p.] (2017).
[3] Kh. P. Gnatenko *Evaluation of variational quantum states entanglement on a quantum computer by the mean value of spin* arXiv:2301.03885 (2023).
[4] Ch. Zoufal, Lucchi, A. Woerner, S. *Quantum Generative Adversarial Networks for learning and loading random distribution* npj Quantum Information **5**, 103 (2019).
[5] Kh. P. Gnatenko, N. A. Susulovska *Geometric measure of entanglement of multi-qubit graph states and its detection on a quantum computer*, EPL (Europhys. Lett.) **136**, No. 4, 40003 (2021).
[6] IBM Q experience. https://quantum-computing.ibm.com/
[7] Rigetti Computing: Quantum Computing https://www.rigetti.com/

(←)

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Landau and Bloch theorems for planar harmonic functions

Abstract. In this presentation we discuss a Landau type theorem for properly normalized bounded planar harmonic functions. We also present Bloch type theorems for properly normalized quasiregular harmonic functions and for open harmonic functions.

Keywords and phrases: planar harmonic functions, Landau theorem, Bloch theorem

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Representations of solutions of Lamé system with real coefficients via monogenic functions in the biharmonic algebra

Abstract.

Consider a Lamé (Lamé-type) system

$$\begin{cases} \Delta u(x,y) + p\,\frac{\partial\theta(x,y)}{\partial x} = 0, \\ \Delta v(x,y) + p\,\frac{\partial\theta(x,y)}{\partial y} = 0 \ \forall (x,y)\in D, \end{cases} \tag{1}$$

where $\Delta := \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $p \in \mathbb{R}\setminus\{0\}$, $\theta := \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$.

By a solution of the Lamé system (1) we mean its *regular* solution, i.e., a pair of twice continuously differentiable functions (u, v), which satisfies the equality (1). If $p = -1$ we assume that $\Delta u = \Delta v = 0$ additionally. For some positive values of p the system (1) is a Lamé system of equilibrium equations of isotropic plane theory of elasticity with respect to the vector of displacements $(u(x, y), v(x, y))$.

Among all two-dimensional algebras of the second rank with unity e over the field of complex numbers $\mathbb{C}$ we found a *biharmonic* algebra $\mathbb{B} := \{c_1 e + c_2\rho : c_k \in \mathbb{C}, k = 1, 2\}$, $\rho^2 = e$, containing bases $\{e_1, e_2\}$, such that a general solution of (1) can be rewritten with the help of components of $\mathbb{B}$-valued "analytic" functions $\Phi_k(xe_1 + ye_2)$ (x, y are real variables), $k = 1, 2$, and certain polynomials with the order less or equal to three.

References

[1] Gryshchuk S.V., *Commutative Complex Algebras of the Second Rank with Unity and Some Cases of Plane Orthotropy I.*, Ukr. Math. J., Volume 70, Issue 8 (2019), pp. 1221-1236.

[2] Gryshchuk S.V., *Commutative Complex Algebras of the Second Rank with Unity and Some Cases of Plane Orthotropy II*, Ukr. Math. J., Volume 70, Issue 10 (2019), pp. 1594-1603.

[3] Gryshchuk S.V., *$\mathbb{B}_0$-valued monogenic functions and their applications to the theory of anisotropic plane media*, In: *Analytic Methods of Analysis and Differential Equations: AMADE 2018* (Eds. S.V. Rogosin and M.V. Dubatovskaya), Cambridge: Cambridge Scientic Publishers Ltd, (2020), 33 – 48. ISBN (paperback): 978-1-908106-65-0.

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Physical interpretation of the membership to Kaplan classes of certain functions

Abstract. The presented results concern a certain class of finite products of the form

$$\mathbb{D} \ni z \mapsto F_n(z;T_n;P_n) := \prod_{k=1}^{n} \left(1 - z\mathrm{e}^{-\mathrm{i}t_k}\right)^{p_k} \quad , \tag{1}$$

where $\mathbb{N} \ni n \mapsto T_n := (t_1, t_2, \ldots, t_n)$ is an increasing sequence of values from $[0; 2\pi)$ such that $t_1 := 0$ and $\mathbb{N} \ni n \mapsto P_n := (p_1, p_2, \ldots, p_n)$ is a sequence of real numbers of the same sign. The first results for polynomials with all zeros on unit circle $\mathbb{T}$ (when P_n is a sequence of natural numbers) were given by Jahangiri [4] in terms of a gap condition. Complete membership to Kaplan classes for the polynomials was presented in [1]. In [2] the authors carried out complete membership to Kaplan classes of finite products of the form similar to (1), but with zeros symmetrically situated in $\mathbb{T}$. The presented results from [3] generalize ones from [1] and [2]. The interpretation of the obtained gap condition in terms of mass and density is given in the case where all zeros of the studied functions are situated in $\mathbb{T}$. An open problem is a more sophisticated physical interpretation that allows for the consideration of zeros outside the unit circle.

Keywords and phrases: Kaplan classes, critical points, mass, density

Joint work with: Maciej Parol

References

[1] S. Ignaciuk, M. Parol, *Zeros of complex polynomials and Kaplan classes*, Analysis Mathematica **46** (2020), 769–779. DOI: https://doi.org/10.1007/s10476-020-0044-8.

[2] S. Ignaciuk, M. Parol, *Kaplan classes of a certain family of functions*, Annales UMCS Sectio A Mathematica **74(2)** (2020), 31-40. DOI: https://doi.org/10.17951/a.2020.74.2.31-40.

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Non-linear electrodynamics derived from the Kaluza-Klein 5-dimensional gravity

Abstract. The lagrangian of the Kaluza-Klein theory, in its simplest five-dimensional version, should include not only the scalar curvature R, but also the quadratic Gauss-Bonnet invariant.

The metric tensor of the five-dimensional Kaluza-Klein space displays 15 degrees of freedom, includong the space-time metric (10 components), the electromagnetic 4-potential (4 components) and the scalar field represented by the g_55 component. We consider the case where only electromagnetic potential is present on the Minkowskian space-time, and in the absence of scalar field.

General lagrangian is computed and the resulting non-linear equations which generalize Maxwell's system in a quite unique way are investigated A possibility of the existence of static solutions is presented, and the qualitative behaviour of such solutions as models for the electron is discussed.

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The grad div operator

Abstract. In this presentation we investigate some natural differential operators in the bundle of symmetric tensors on a Riemannian manifold M of dimension n. In particular we investigate two operators: the gradient and the divergence. The definition of these two operators is analogous to the definition of gradient and divergence in the bundle of skew-symmetric forms given by H. M. Rummler [2]. It is interesting that the composition of the divergence and the gradient led to a second order linear differential operator div grad that was investigated in detail in the author's PhD dissertation [1]. However, the subject of our research here will be the grad div operator.

Keywords and phrases: Symmetric tensors, symmetric derivative, the gradient, the divergence.

References

[1] A. Kimaczyńska, *The differential operators in the bundle of symmetric tensors on a Riemannian manifold*, PhD thesis, Łódź University, Faculty of Mathematics and Computer Science, Łódź (2016), 59 pp;
[2] H. M. Rummler, *Differential forms, Weitzenböck formulae and foliations*, Publicacions Matematiques 33 (1989), 543–554.

(←)

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Different methodologies in description Classical and Quantum Conway Game of Life

Abstract. Classical stochastic Conway Game of Life is expressed by the dissipative Schrödinger equation and dissipative tight-binding model. This is conducted at the prize of usage of time dependent anomalous non-Hermitian Hamiltonians as with occurrence of complex value potential that do not preserve the normalization of wave-function and thus allows for mimicking creationism or annihilationism of cellular automaton. Simply saying time-dependent complex value eigenenergies are similar to complex values of resonant frequencies in electromagnetic resonant cavities reflecting presence of dissipation that reflects energy leaving the system or being pumped into the system. At the same time various aspects of thermodynamics were observed in cellular automata that can be later reformulated by quantum mechanical pictures. The usage of Shannon entropy and mass equivalence to energy points definition of cellular automata temperature. Contrary to intuitive statement the system dynamical equilibrium is always reflected by negative temperatures. Diffusion of mass, energy and temperature as well as phase of proposed wave function is reported and can be directly linked with second thermodynamics law approximately valid for the system, where neither mass nor energy is conserved. The concept of complex-valued mass mimics wave-function behavior. Equivalence an anomalous second Fick law and dissipative Schrödinger equation is given. Dissipative Conway Game of Life tight-binding Hamiltonian is given using phenomenological justification.

(←)

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The application of unsupervised learning to the AC susceptibility measurements of High-Temperature Superconductors

Abstract. Machine learning (ML) is the study of computer algorithms that improve automatically through experience. ML algorithms are built on a mathematical model based on data, in order to make predictions or decisions without being explicitly programmed to do so. Unsupervised learning (UL) is a subfield of ML. UL algorithms look for previously undetected patterns in a dataset with no preexisting labels and with a minimum of human supervision. Great progress has been made in a quest to discover, develop or rene various machine learning algorithms in recent years and new ways of data analysis have been shown. The ML application to the analysis of datasets is a state of the art technique which allows to make breakthroughs in various areas of science and engineering.

Our work aims to provide a rst insight into application of clustering techniques to the large dataset of AC susceptibility measurements of High-Temperature Superconductors (HTS). It should allow recovering known relationships between dierent types of HTS and their superconducting properties.

We show that it is possible to represent the most signicant features of a single AC measurement of a HTS sample as 5 numerical values by using a Convolutional 1D Autoencoder and the Bag Of Words model. The most distant 5D representations of χ(T) are for samples, which have the most different superconducting properties i.e. thin layer HTS and grinded and pressed polycrystalline HTS so the 5D representation of the χ(T) dataset preserves the most important features of the measurement of the HTS sample. However the cluster analysis of the 5D (T) dataset by two clustering algorithms did not reveal the existence of clearly distinct classes of χ(T) measurements. Though a t-SNE visualisation (e.g.1) in 3D space shows that some clustering exists and part of the measurements are mainly arranged on some sort of cluster boundary. Therefore, more advanced analysis could be performed.

(←)

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Iterative construction of discrete Laplacians on 2D square and hexagonal lattices using BTBB and other sequences

Abstract. The BTBB construction discussed in [1, 2, 3, 4] and its various applications were given in mathematics, biology, chemistry, knot theory, formal language theory. Several papers have been written on "Binary and ternary structures of the evolutions in the universe". Parts I-V by Suzuki, pointing out the uniqueness of this sequence.

We want to demonstrate iterative construction of discrete Laplacians on 2D square and hexagonal lattices using BTBB sequence and compare with the constructions obtained with the use of other sequences. The BTBB sequence occurs to be exceptional under many respects.

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The Koebe radius for certain class of polynomials

Abstract. The presented results concern close-to-convex polynomials with all zeros of derivative in the unit circle $\mathbb{T}$. The minimal disc containing all images of the unit disc $\mathbb{D}$ and the maximal disc contained in all images of $\mathbb{D}$ through polynomials of degree 3 and 4 were determined in [4]. Moreover, all extremal functions for both problems were received. The same problem for polynomials of degree 5 was solved in [5]. In addition, the hypothesis for polynomials of odd degree was also given.

Joint work with: Szymon Ignaciuk

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Does the convex Peano curve exist and what can we say about operators?

Abstract. We present an unexpected construction of continuous surjection $f : [0,1] \to T$ for a given closed compact set T in $\mathbb{R}^2$. Some important consequences are connected with convexity preserving operators.

(←)

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Monogenic functions in infinite-dimensional vector spaces with a commutative multiplication and harmonic vectors

Abstract. We consider special topological vector spaces with a commutative multiplication for some of elements of the spaces. The consideration of such topological vector spaces is motivated by the need to describe all spatial harmonic functions as components of hypercomplex monogenic functions. Monogenic functions are understood as continuous and differentiable in the sense of Gâteaux functions. We prove that all spatial harmonic functions are components of monogenic functions taking values in the mentioned spaces. We describe relations between the mentioned monogenic functions and harmonic vectors in the three-dimensional real space and establish sufficient conditions for infinite monogeneity of functions. Unlike the classical complex analysis, it is done in the case where the validity of the Cauchy integral formula for monogenic functions remains an open problem.

(←)

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Towards construction of analog solver of Schrödinger equation based on Kron's second model

Abstract. The analog electronic computers are a type of circuitry used to calculate specific problems using the physical relationships between the voltages and currents in circuits. One specific class of these circuits are computers based on the interactions between passive circuit elements. Models presented by Gabriel Kron are the example of using such passive elements to construct a solver for the problem of free quantum particles entering the well of potential. The aim of this experiment is construction of the netlist based simulation for Kron's second model and test the possibility of its generalisation for the different shapes of potential, starting with the well known harmonic potential. In the long term, this opens the possibility of creating the high efficiency hybrid computers capable of simulating the complex quantum systems, such as the case of multiple electrostatically interacting particles, such as the quantum logic gates.

(←)

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Simulation and experiments with superconducting single photon detectors

Abstract. Theory of operation of superconducting single photon detectors is presented and discussed.

(←)

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Universal modeling of electrostatic semiconductor quantum gates of any topology interfaced to Josephson junction quantum circuit

Abstract. Single electron devices implemented in the chain of coupled quantum dots become quite promising ways of implementation of qubits [1], quantum logical gates and quantum communication systems due to usage of well-developed CMOS technology that guarantees very high integration. On another hand the superconducting circuits achieved limited level of scalability and are the less noisy integrated systems at low temperatures, so they account for scalable superconducting qubits. High level of CMOS scalability and low noise level in case of Josephson junction makes it necessary to consider the hybrid superconducting-semiconductor quantum devices and interfaces between them as given by [2]-[3]. The concept of programmable quantum matter and thus quantum circuits can be modeled by usage of quasi one-dimensional models of semiconductor (superconducting) nanowires in Schrödinger (Bogoliubov-de Gennes) or tight-binding formalisms. In such a case, open loop nanowires of arbitrary topology can be approximated by quasi one-dimensional description. The presented scheme can be easily generalized to N interacting electrons placed at N different semiconductor nanowires, whose functionality can be regulated with proper external biasing electric and electromagnetic fields. The interaction of Josephson junction with semiconductor quantum dots is described both by capacitive or inductive channels. In such a way, quantum information processing can be studied in dependence on different topologies of semiconductor nanowires in various electromagnetic conditions.

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(←)

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$\mathcal{Q}_p$ spaces for hyperholomorphic functions in the unit ball of $\mathbb{R}^4$

Abstract. The classical $\mathcal{Q}_p$ for $1 \leq p < \infty$ spaces were introduced by R. Aulaskari and P. Lappan in [1] and for $0 < p \leq 1$ by R. Aulaskari et al in [2]. The hyperholomorphic generalization to the unit ball of $\mathbb{R}^3$ was given by K. Gürlebeck, et al in [3].

Consider the unit ball of $\mathbb{B} \subset \mathbb{R}^4$ and the set of real quaternions $\mathbb{H}$. For $0 \leq p < 2$, the $\widetilde{\mathcal{Q}}_p(\mathbb{B})$ space consists of the $\mathbb{H}$ right-module of hyperholomorphic functions $f : \mathbb{B} \to \mathbb{H}$ satisfying

$$\| f \| = \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\overline{D} f(x)|^2 g^p(x,a)\, dx < \infty,$$

where $g(x,a) = \frac{1}{|\varphi_a(x)|^2} - 1$ is the composition of a modified fundamental solution of the Laplacian in the unit ball of $\mathbb{B}$, that is, $\frac{1}{|x|^2}$ and the Möbius transformation $\varphi_a(x) = (a - x)(1 - \bar{a}x)^{-1}$.

Let $0 \leq p < \infty$. The $\mathcal{Q}(\mathbb{B})$ space consists of the $\mathbb{H}$ right-module of hyperholomorphic functions $f : \mathbb{B} \to \mathbb{H}$ satisfying

$$\sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\overline{D} f(x)|^2 (1 - |\varphi_a(x)|^2)^p\, dx < \infty.$$

In this talk we study several properties of these and another related spaces.

Keywords and phrases: quaternion, Bloch, Dirichlet and $\mathcal{Q}_p$-spaces,

Joint work with: L. M. Tovar S. (ESFM-IPN-MEX)

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(←)

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Quaternionic foundations of Quantum Mechanics and spin 1/2 visualization

Abstract. This talk explores the intriguing link between quaternion quantum mechanics and the model of the elastic continuum proposed by Augustin Cauchy, highlighting the potential of this interdisciplinary connection to advance our understanding of quantum physics. We elucidate the derivation of non- and relativistic wave equations from the Cauchy theory and Hamilton quaternion algebra, demonstrating how the quaternion representation of the elastic continuum provides a comprehensive picture of quantum mechanics. This innovative interpretation addresses the ontological challenges often associated with quantum theory, offering a means to generalize and test the theory.

Transitioning from the theoretical to the practical, we delve into a unique visualization of spinors using quaternions. We elucidate how the quaternionic representation enables a comprehension of the Dirac equation and the concept of spin 1/2, pivotal to the Planck-Kleinert model and the Pauli Exclusion Principle. We present an interactive simulation of a periodically twisting 3D grid, visualizing the spin 1/2 behavior of fermions in an elastic solid.

By visualizing Dirac spinors using two orthogonal quaternions, we illuminate the complex rotation behavior of spin 1/2 particles, drawing analogies with the famous Dirac belt trick. We define two quaternion functions that represent rotations as functions of time and distance, respectively. The visualization of the resultant spinor function as a series of concentric shells offers a illuminating representation of quantum spin, enhancing our understanding of its behavior in both time and space.

Keywords and phrases: quaternion

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Approximation of classes of Poisson integrals by Fejér means

Abstract. Asymptotic equality for upper bounds of deviations of Fejér means on classes $\mathrm{G}(q)$ was obtained in [1] (see definitions):

$$\mathcal{E}\left(\mathrm{G}(q);\sigma_n[f]\right) := \sup_{f\in\mathrm{G}(q)} \|f(\cdot)-\sigma_n[f](\cdot)\|_{\mathrm{C}} = \frac{4q}{\pi n(1+q^2)} + O(1)\frac{q^n}{n}, \quad q\in(0;q_0], \tag{1}$$

where $q_0 = \sqrt{2+\sqrt{5}-2\sqrt{2+\sqrt{5}}} \approx 0.346$, $O(1)$ is a quantity uniformly bounded with respect to n. Also, in [1] may be found an overview of the literature. The aim of this work is to present asymptotic equality for upper bounds of deviations of Fejér means taken over classes of Poisson integrals in case $q\in[q_0;1)$.

Theorem. *Let $f\in\mathrm{G}(q)$. For $q\in[q_0;1)$ the equality hold as $n\to\infty$*

$$\mathcal{E}\left(\mathrm{G}(q);\sigma_n[f]\right) = \frac{2}{\pi n}\frac{(1+q^2)^2}{(1-q^2)\left(1-q^2+\sqrt{2(1+q^4)}\right)} + O(1)\frac{q^n}{n(1-q)^3}, \tag{2}$$

where $O(1)$ is uniformly bounded with respect to n, q.

Keywords and phrases: Poisson integral, Fejér mean; asymptotic equality

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On the exponential asymptotic of solutions of the nonlinear Cauchy-Riemann-Beltrami type equation

Abstract. Let G be a domain in a complex plane $\mathbb{C}$, so a connected and open subset $\mathbb{C}$, and let $\mu\colon G\to\mathbb{C}$ be a measurable function with $|\mu(z)|<1$ a.e. (almost everywhere) in G. *Beltrami equation* is called an equation of the form

$$f_{\overline{z}}=\mu(z)f_z\,, \tag{1}$$

where $f_{\overline{z}}=\frac{1}{2}(f_x+if_y)$, $f_z=\frac{1}{2}(f_x-if_y)$, $z=x+iy$, f_x and f_y are partial derivatives of the mapping f of x and y, respectively.

Let $\sigma\colon G\to\mathbb{C}$ be a measurable function and $m\geqslant 0$. Consider in the polar coordinate system (r,θ) the following equation:

$$f_r=\sigma(re^{i\theta})\,|f_\theta|^m\,f_\theta, \tag{2}$$

where f_r and f_θ are partial derivatives of the mapping f of r and θ, respectively. Considering the formulas $rf_r=zf_z+\overline{z}f_{\overline{z}}$, $f_\theta=i(zf_z-\overline{z}f_{\overline{z}})$, the equation (2) may be written in a complex form:

$$f_{\overline{z}}=\frac{z}{\overline{z}}\,\frac{\sigma(z)\,|z|\,i|zf_z-\overline{z}f_{\overline{z}}|^m-1}{\sigma(z)\,|z|\,i|zf_z-\overline{z}f_{\overline{z}}|^m+1}\,f_z. \tag{3}$$

The mapping $f\colon G\to\mathbb{C}$ is called *regular at the point* $z_0\in G$, if f has a complete differential and its Jacobian $J_f=|f_z|^2-|f_{\overline{z}}|^2\neq 0$ at this point. The homeomorphism f of the Sobolev class $W^{1,1}_{\rm loc}$ is called *regular* if $J_f>0$ a.e. *A regular homeomorphic solution* of the equation (3) is called regular homeomorphism $f\colon G\to\mathbb{C}$, that satisfies the equation (3) a.e. in G.

Later on we use the following notations

$$\gamma_r=\{z\in\mathbb{C}:|z|=r\}\,,\quad \mathbb{A}(0,r_1,r_2)=\{z\in\mathbb{C}:\,r_1<|z|<r_2\},\quad \mathbb{B}=\{z\in\mathbb{C}:|z|<1\}.$$

Theorem. *Let $m>0$, $f\colon\mathbb{B}\to\mathbb{C}$ be a regular homeomorphic solution of the equation (3) of the Sobolev class $W^{1,2}_{\rm loc}$ with the normalization $f(0)=0$ and $\int\limits_{\gamma_t}\frac{ds}{\left(\operatorname{Im}\overline{\sigma(z)}\right)^{\frac{1}{m+1}}}\neq\infty$ for almost all $t\in(0,1)$. Suppose that for some numbers $C>0$, $p>0$, $0\leqslant\alpha\leqslant\frac{m+2}{m+1}$ and $\varepsilon_0\in(0,1)$ the following condition is fulfilled*

$$\int\limits_{\mathbb{A}(0,\,\varepsilon,\,\varepsilon_0)}\frac{\Phi_p^{\frac{m+2}{m+1}}(|z|)dxdy}{|z|\left(\operatorname{Im}\overline{\sigma(z)}\right)^{\frac{1}{m+1}}}\leqslant C\,e^{\frac{\alpha}{\varepsilon^p}}\,,$$

for any $\varepsilon\in(0,\varepsilon_0)$, where $\Phi_p(t)=\frac{e^{\frac{1}{t^p}}}{t^{p+1}}$. Then

$$\liminf_{z\to 0}|f(z)|e^{\frac{\beta}{|z|^p}}\leqslant c_0\,p^{\frac{m+2}{m}}C^{\frac{m+1}{m}}<\infty,$$

where $\beta=\frac{m+2-\alpha(m+1)}{m}$ and $c_0=(2\pi)^{-\frac{m+1}{m}}m^{-\frac{1}{m}}$.

Keywords and phrases: Beltrami equation, regular homeomorphic solution of the Beltrami equation

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(←)

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Influence of quantized vortex dynamics on superconducting electronics

Abstract. Superconducting materials are currently the subject of interest in modern electronic technologies, especially because they exhibit quantum phenomena on a macroscopic scale. The most evident examples are quantized vortices transporting magnetic flux, which movement determines the macroscopic current carrying phenomena in the superconducting materials, used in superconducting electronics. In the communication will be discussed the influence of the movement of the pancake type vortices in HTc multilayered superconductors in the flux creep process on their current-voltage characteristics and then critical current density, which are the essential parameters characterizing the superconducting electronic quantum devices, as well as power superconducting apparatus, as superconducting magnets with current leads. The authors works related to the superconducting cables will be reviewed shortly too. In presentation will be discussed how various initial states of the captured vortices and their shape influence the pinning potential barrier and then current-voltage characteristics. It will be presented how fitting the current-voltage characteristics to experimental data in the model allows to determine the material parameters of the HTc superconductors such as nano-sized defects concentration. Next critical current density of the superconducting materials in the vortex dynamics model will be analyzed as the function of the irradiation dose creating the nano-sized defects for various values of the magnetic induction. Initial enhancement of the critical current results from the new pinning centers creation during irradiation. At the same time, the characteristic maximum of this dependence and next decrease is interpreted as related to the destructive action on the crystal lattice of the high dose of irradiation. The vortex dynamics analysis of HTc superconductors is applied for the interpretation of the dynamic anomalies of the current-voltage characteristics in a slowly varying magnetic field, which effect can be used in superconducting electromagnetic sensors, as well as for the description of the irradiation from long Josephson's junctions with rotating vortex between the Josephson's junctions covers. Finally, the results of the previous investigations of the superconducting semiconductors with vacancies of lanthanum $La_{3-x}Vac_xSe_4$ will be discussed, possible class of the materials useful for the construction of the joined superconducting – semiconducting electronic quantum devices.

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On some exact solutions of certain nonlinear partial differential equations

Abstract. In this presentation some exact solutions (the so-called functionally-invariant solutions) of certain nonlinear partial differential equations (some heavenly equations and Landau-Lifshitz equation), will be presented. The functionally-invariant solutions were found by first time in [3]. The results presented in this talk, are obtained using the so-called structural decomposition method, which was presented and applied in [4], [5], [6], [7], [8], [9]. This method differs from the methods applied by other Authors, for e.g. [1].

Keywords and phrases: [functionally-invariant solutions, heavenly equations, Landau-Lifshitz equation]

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Hybrid Schrödinger-Ginzburg-Landau (Sch-GL) approach in study of superconducting integrated structures

Abstract. Various superconducting lattices were simulated and can be treated as lattices of superconducting atoms with preimposed symmetry in 1, 2 and 3 dimensions. Hybrid Schrödinger-Ginzburg-Landau approach is based on the fact of the mathematical similarity of Ginzburg-Landau (GL) and Schrödinger formalisms. Starting from Schrödinger approach by change of term $V(x) - E$ with term $\alpha(x) + \beta(x)|\psi(x)|^2$ we arrived at the Ginzburg-Landau equation. In the presented relaxation algorithm we use one and two dimensional ground energy solutions of Schrödinger equation and placed them as starting trial solution for GL relaxation method. In consecutive steps we increase the nonlinear term in the GL equation which results in achieving a stable approach of solution of GL equation. The obtained numerical results and used methodology form simulation platform bases for study of superconducting integrated structures that can model various superconducting devices. In general, one can model time-dependent geometry of superconducting structures.

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Binary and ternary structures in evolution theory (VI)
An entropic description of the evolution of the universe
(Open discussions)

Abstract. A method of entropy and non-commutative Galois theory for the evolution of the universe is presented. Open discussions will be expected. New ideas are wanted

Joint work with: Julian Ławrynowicz, Małgorzata Nowak-Kępczyk

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The full set of invariants characterizing the $Sp(n)$-orbits in the real Grassmannians $G^{\mathbb{R}}(k,4n)$

Abstract. The purpose of this work is the determination of the full set of invariants for the $Sp(n)$-orbits in the real Grassmannians $G^{\mathbb{R}}(k,4n)$ of the k-dimensional subspaces of a real $4n$-dimensional vector space V. We endow V with an Hermitian quaternionic structure $(\mathcal{Q}=(I,J,K)_{\mathbb{R}},<,>)$ where $\mathcal{H}=(I,J,K)$ is an Hypercomplex structure made of three anticommuting complex structures satisfying $IJ=K$ and $<,>$ is an Hermitian scalar product i.e. a positive definite scalar product w.r.t. which (I,J,K) are skew-symmetric.

In the previous articles on the same subject we considered and solved the same problem for 2-dimensional subspaces ([16]), complex and Σ-complex (defined as orthogonal sum of complex subspaces by different compatible complex structures) subspaces [18], isoclinic subspaces ([19]) (among which the special cases of quaternionic and real Hermitian product subspaces). We recall that a subspace U is isoclinic if all the principal angles of the pair (U,AU) are the same for any compatible complex structure $A\in(Q)$. Anyway the problem was still open for a generic subspace $U\subset V$.

Here we give the solution for the general case. To this aim we first prove that, if a subspace U is isoclinic with respect to (I,J,K), then U is isoclinic. We show that, fixed a compatible complex structure, say $I\in\mathcal{H}$, any $U\subset V$ admits a canonical I-decomposition into the orthogonal sum of uniquely defined isoclinic subspaces. Therefore the determination of the $Sp(n)$-orbit of U follows from the theory of isoclinic subspaces which appear in ([19]). There we showed that the set made of the principal angles $(\theta^I,\theta^J,\theta^K)$ and the cosines (ξ,χ,η,Δ) of the angles between some pairs of special vectors determines the $Sp(n)$-orbit of an isoclinic subspace U. Then the union of such sets, each one referred to an isoclinic addend, determine the full set of invariants characterizing the $Sp(n)$-orbit of the k-dimensional subspace U in the real Grassmannian $G^{\mathbb{R}}(k,4n)$.

Keywords and phrases: Hermitian quaternionic structure, principal angles, isoclinic subspaces

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Emergence of topological defects in finite 2-D lattices during second phase order transitions

Abstract. In this work we study the Kibble-Zurek [1] mechanism that describes the emergence of topological defects in finite 2-D lattices during second phase order transitions. We will discuss the numerical method used to recreate the process, mention the behaviour of 1-D atomic chains and expound on the difficulty behind the categorization of the 3 different types of defects (point, 1-D and 2-D) when using statistical methods. We will shortly discuss the mathematics concerning both the nature of the defects and the numerical process (Euler-Maruyama Leapfrog).

Our work is based on previous work that is done concerning atomic chains [2]-[5].

Joint work with: Hariton Polatoglou

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Continuous mean periodic extension of functions from a segment

Abstract. Let $n \geq 2$, $\mathcal{E}'(\mathbb{R}^n)$ be the space of compactly supported distributions on $\mathbb{R}^n$. We study the following version of the mean periodic extension problem.

(i) Suppose that $T \in \mathcal{E}'(\mathbb{R}^n)$, and E is a non-empty closed subset of $\mathbb{R}^n$. What conditions guarantee that for a function $f \in C(E)$ there is a function $F \in C(\mathbb{R}^n)$ coinciding with f on E and satisfying the convolution equation $F * T = 0$ in $\mathbb{R}^n$?

(ii) If such an extension F exists then estimate the growth of F at infinity.

We present a solution of this problem for a broad class of distributions T in the case when E is a segment in $\mathbb{R}^n$.

For results related to the continuation of mean periodic functions, see [1]-[3] and the references there.

Keywords and phrases: Convolution equations, mean periodicity, spherical transform, quasi-analyticity

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Discovery of polarization degree of freedom for localized light

Abstract. The spectrum of electromagnetic plane waves in any isotropic homogeneous medium is twice degenerate with respect to polarization at any frequency and in any direction, because the dispersions of TE- and TM-polarized eigenmodes are absolutely the same. The operational principle of any bulk classical polarizer is based on the removal of the polarization degeneracy by using an anisotropic slab.

Miniaturization and planar technologies lead to the high localization of the electromagnetic signal in the plane of propagation. However, at the same time, the degeneracy is removed. The simplest example is the dielectric slab waveguide demonstrating the propagation of highly localized guided modes with different dispersions of TE and TM modes. So, there is no polarization degree of freedom for planar photonic devices with in-plane electromagnetic wave propagation, since the TE and TM localized modes are not degenerate. It significantly limits the functionality of flat optical and planar photonic devices. In particular, the polarizer of guided waves cannot be implemented without the broadband TE-TM polarization degeneracy.

In this work, we show the possible resolution of this fundamental problem via the nanostructuring of the dielectric slab. We aim merging the polarization degeneracy and high localization for guided waves propagating within an all-dielectric metasurface. Namely, we theoretically study the periodic subwavelength array of cylinders with high refractive index and demonstrate the broadband TE-TM polarization degeneracy of its guided modes. This concept can be transferred to the optical and infrared ranges using other material platforms and opens new opportunities for the polarization control of guided waves such as metasurface-based waveguide polarizer [1].

Another possible implementation includes the self-complementary metasurfaces obeying Babinet's duality principle [2]. In this case, we excite independently the TE- and TM-polarized surface waves at self-complementary metasurfaces. Then, by adjusting the complex amplitudes of TE and TM eigenmodes, we demonstrate the excitation of surface waves of linear, circular and arbitrary elliptical polarization [3].

These results could potentially become a platform for new generation of planar photonic polarization devices.

Keywords and phrases: photonics, polarization, metasurfaces

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Local systems management of different energy sources

Abstract. The question arises: Is the correct solution for energy information system having container controlling heat storage, electric energy sources that are unstable or diverse in time? The goal is to obtain high-quality energy from OZE as from energetic system grid or burning methane or other fuels. The key to success is set as the miniaturization of consumption energy for device AGD. Is it necessary to turn down things in the house that shouldn't be on power all day? The presented method is the first step in distributed energy networks (RSE), adaptation to smart house technology, and management strategy for controlling of nodes of energy supply system. In such a way a lot of houses in village/city have creative cluster storage energy system. This solution reduces the load on PSE. Parallel to PSE working local direct power lines, alternative lines are used to control flow energy from OZE to storage devices. The presentation gives not very new concepts, but provides a specific diagram of the operation of a high-power system about 1MW.

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Application of Scattering Parameters to The DPL Time-Lag Parameter Estimation using MEMS Structures

Abstract. The dedicated micro-electro-mechanical system (MEMS) test structure has been developed and fabricated for material parameters' estimation for the dual-phase-lag (DPL) model at the nanoscale in modern integration circuit (IC) structures [1, 2]. The temperature time-lag estimation procedure of dielectrics is presented based on the electric scattering parameters measured by a vector network analyser for the considered MEMS test structure together with the 2-omega method [2]. The proposed methodology has the ability to estimate the time-lag parameter with high accuracy and is also suitable for the temperature time-lag estimation for other manufacturing process technologies of ICs for insulation materials such as silicon dioxide (SiO2), silicon nitride (Si3N4), titanium nitride (TiN), and hafnium dioxide (HfO2).

Keywords and phrases: dual-phase-lag heat transfer model; no-mesh FDM; S-parameters measurements; scattering parameters; DPL time lag estimation; multi-domain modelling; material parameter estimation; heat transfer at the nanoscale

Joint work with: Z. Kulesza, M. Jankowski, A. Napieralski

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On the role of the point at infinity in Deny's principle of positivity of mass for Riesz potentials

Abstract. This talk is based on [9], and it deals with the theory of potentials on $\mathbb{R}^n$, $n \geqslant 2$, with respect to the Riesz kernel $|x-y|^{\alpha-n}$, $\alpha \in (0,2]$, $\alpha < n$, where $|x-y|$ is the Euclidean distance between $x, y \in \mathbb{R}^n$. Denote by $\mathfrak{M}^+$ the cone of all positive Radon measures μ on $\mathbb{R}^n$ such that the Riesz *potential*

$$U^\mu(x) := \int |x-y|^{\alpha-n}\, d\mu(y)$$

is not identically infinite on $\mathbb{R}^n$, which according to [5, Section I.3.7] occurs if and only if

$$\int_{|y|>1} \frac{d\mu(y)}{|y|^{n-\alpha}} < \infty.$$

Then U^μ is actually finite everywhere on $\mathbb{R}^n$, up to a set of *zero Riesz capacity*, cf. [5, Section III.1.1].

The principle of positivity of mass was first introduced by J. Deny (see e.g. [2]), and for Riesz potentials it reads as follows [3, Theorem 3.11].

Theorem 1. *For any $\mu, \nu \in \mathfrak{M}^+$ such that*

$$U^\mu \leqslant U^\nu \quad \text{everywhere on } \mathbb{R}^n, \tag{1}$$

we have $\mu(\mathbb{R}^n) \leqslant \nu(\mathbb{R}^n)$.

It is easy to verify that (1) can be slightly weakened by replacing 'everywhere on $\mathbb{R}^n$' by 'nearly everywhere on $\mathbb{R}^n$' (see [8, Theorem 2.6], establishing the principle of positivity of mass for potentials with respect to rather general function kernels on locally compact spaces). Recall that a proposition $\mathcal{P}(x)$ is said to hold *nearly everywhere* (*n.e.*) on $A \subset \mathbb{R}^n$ if $c_*(E) = 0$, where E is the set of all $x \in A$ for which $\mathcal{P}(x)$ fails, while $c_*(E)$ denotes the *inner Riesz capacity* of E, see [5, Section II.2.6].

The main result of this talk, given by Theorem 2, shows that Theorem 1 still holds even if (1) is fulfilled only on a proper subset A of $\mathbb{R}^n$, which however must be 'large enough' in an arbitrarily small neighborhood of $\infty_{\mathbb{R}^n}$, the Alexandroff point of $\mathbb{R}^n$. This discovery illustrates a special role of the point at infinity in Riesz potential theory, in particular with regard to the principle of positivity of mass.

Theorem 2. *Given $\mu, \nu \in \mathfrak{M}^+$, assume there exists $A \subset \mathbb{R}^n$ which is not inner α-thin at infinity, and such that*

$$U^\mu \leqslant U^\nu \quad \text{n.e. on } A.$$

Then

$$\mu(\mathbb{R}^n) \leqslant \nu(\mathbb{R}^n).$$

Recall that according to [4, 7], $A \subset \mathbb{R}^n$ is said to be *inner α-thin at infinity* if

$$\sum_{k\in\mathbb{N}} \frac{c_*(A_k)}{q^{k(n-\alpha)}} < \infty,$$

where $q \in (1,\infty)$ and $A_k := A \cap \{x \in \mathbb{R}^n :\ q^k \leqslant |x| < q^{k+1}\}$; or equivalently, if either A is bounded, or $x = 0$ is an inner α-irregular boundary point for the inverse of A with respect to $|x| = 1$. (For the concept of inner α-irregular points for arbitrary $A \subset \mathbb{R}^n$ and relevant results, see [6, Section 6]; compare with [5, Section V.1], where A was required to be Borel.) We emphasize that *if A is not inner α-thin at infinity, then necessarily $c_*(A) = \infty$; but not the other way around* (see [7, Section 2]).

The following theorem shows that Theorem 2 is *sharp* in the sense that the requirement on A of not being α-thin at infinity can not in general be weakened.

Theorem 3. *If $A \subset \mathbb{R}^n$ is inner α-thin at infinity, then there are $\mu_0, \nu_0 \in \mathfrak{M}^+$ such that $U^{\mu_0} = U^{\nu_0}$ nearly everywhere on A, but nonetheless, $\mu_0(\mathbb{R}^n) > \nu_0(\mathbb{R}^n)$.*

Nevertheless, Theorem 2 remains valid for *arbitrary* $A \subset \mathbb{R}^n$ once we impose upon $\mu, \nu \in \mathfrak{M}^+$ suitable additional requirements (see Theorem 4 below).

A measure $\mu \in \mathfrak{M}^+$ is said to be *carried* by $A \subset \mathbb{R}^n$ if $\mathbb{R}^n \setminus A$ is μ-negligible, or equivalently if A is μ-measurable and $\mu = \mu|_A$, $\mu|_A$ being the trace of μ to A, cf. [1, Section V.5.7]. We denote by $\mathfrak{M}^+_A$ the cone of all $\mu \in \mathfrak{M}^+$ carried by A. (For closed A, μ is carried by A if and only if it is supported by A.)

A measure $\mu \in \mathfrak{M}^+$ is said to be *C-absolutely continuous* if $\mu(K) = 0$ for every compact set $K \subset \mathbb{R}^n$ of zero Riesz capacity. This certainly occurs if $\int U^\mu \, d\mu$ is finite (or, more generally, if U^μ is locally bounded); but not conversely, see [5, pp. 134–135].

Theorem 4. *For any set $A \subset \mathbb{R}^n$ and any C-absolutely continuous measures $\mu, \nu \in \mathfrak{M}^+_A$ such that $U^\mu \leqslant U^\nu$ n.e. on A, we still have $\mu(\mathbb{R}^n) \leqslant \nu(\mathbb{R}^n)$.*

Remark 1. If $A \cap A_I = \varnothing$, where A_I denotes the set of all inner α-irregular points for A, then the requirement of C-absolute continuity imposed on μ and ν, is unnecessary for the validity of Theorem 4.

Remark 2. The proofs of the above-quoted theorems are based on the theory of inner α-Riesz balayage as well as on that of inner α-Riesz equilibrium measures, both originated in [6, 7] (see also [8]). The concept of inner equilibrium measure is understood in an extended sense where its energy as well as its total mass may be infinite. The following two facts of these theories are crucial to our proofs:

- $A \subset \mathbb{R}^n$ is not α-thin at infinity if and only if the inner balayage of *any* $\mu \in \mathfrak{M}^+$ to A preserves its total mass (see [7, Corollary 5.3]).
- The inner α-Riesz equilibrium measure of $A \subset \mathbb{R}^n$ exists if and only if A is α-thin at infinity (see [7, Theorem 2.1]).

Remark 3. The results presented in the talk have already found applications to minimum Riesz energy problems in the presence of external fields, see for instance [10, Section 4.10].

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