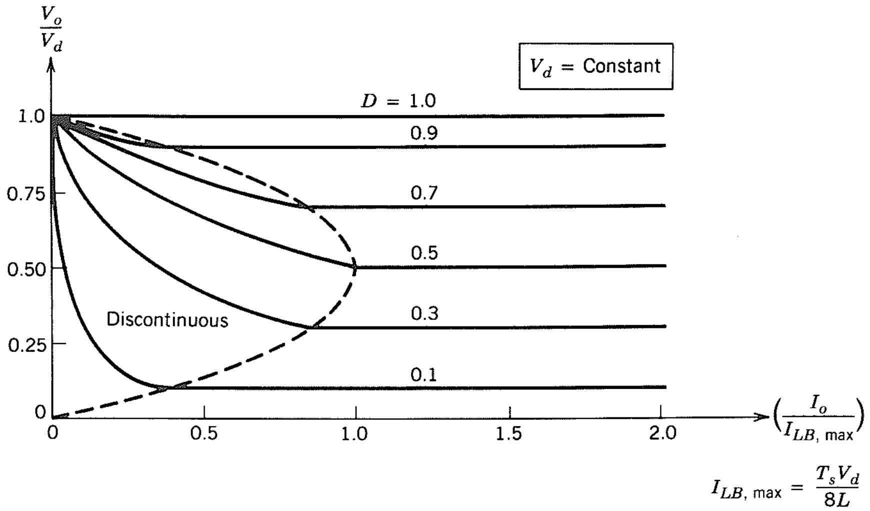

Figure 7-8 Step-down converter characteristics keeping V_d constant.

It should be noted that the operation corresponding to $D = 0$ and a finite V_o is, of course, hypothetical because it would require V_d to be infinite.

From Eqs. 7-18 and 7-19

$$I_{LB} = (1 - D)I_{LB,\max} \tag{7-20}$$

For the converter operation where V_o is kept constant, it will be useful to obtain the required duty ratio D as a function of $I_o/I_{LB,\max}$. Using Eqs. 7-10 and 7-13 (which are valid in the discontinuous-conduction mode whether V_o or V_d is kept constant) along with Eq. 7-19 for the case where V_o is kept constant yields

$$D = \frac{V_o}{V_d}\left(\frac{I_o/I_{LB,\max}}{1 - V_o/V_d}\right)^{1/2} \tag{7-21}$$

The duty ratio D as a function of $I_o/I_{LB,\max}$ is plotted in Fig. 7-9 for various values of V_d/V_o, keeping V_o constant. The boundary between the continuous and the discontinuous mode of operation is obtained by using Eq. 7-20.

7-3-4 OUTPUT VOLTAGE RIPPLE

In the previous analysis, the output capacitor is assumed to be so large as to yield $v_o(t) = V_o$. However, the ripple in the output voltage with a practical value of capacitance can be calculated by considering the waveforms shown in Fig. 7-10 for a continuous-conduction mode of operation. Assuming that all of the ripple component in i_L flows through the capacitor and its average component flows through the load resistor, the shaded area in Fig. 7-10 represents an additional charge ΔQ. Therefore, the peak-to-peak voltage ripple ΔV_o can be written as

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C}\frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2}$$

From Fig. 7-5 during t_{off}

$$\Delta I_L = \frac{V_o}{L}(1 - D)T_s \tag{7-22}$$

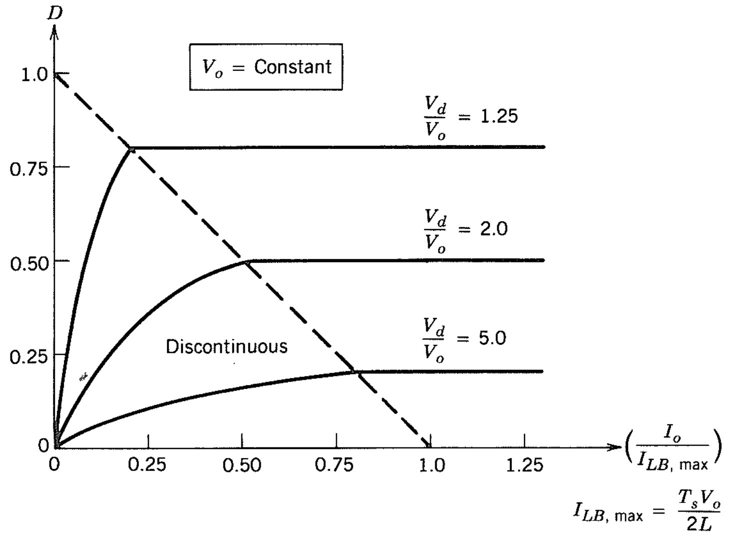

Figure 7-9 Step-down converter characteristics keeping V_o constant.

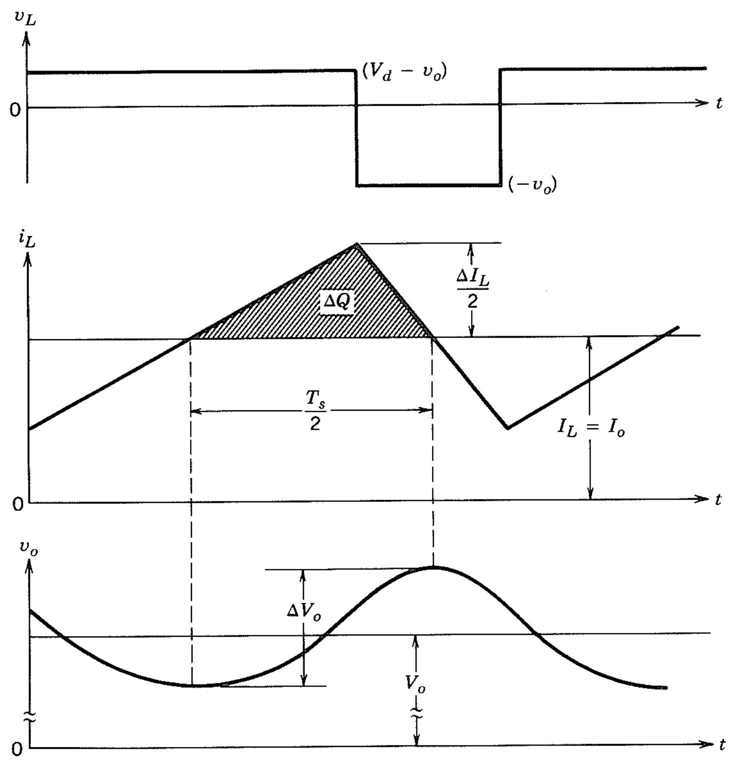

Figure 7-10 Output voltage ripple in a step-down converter.

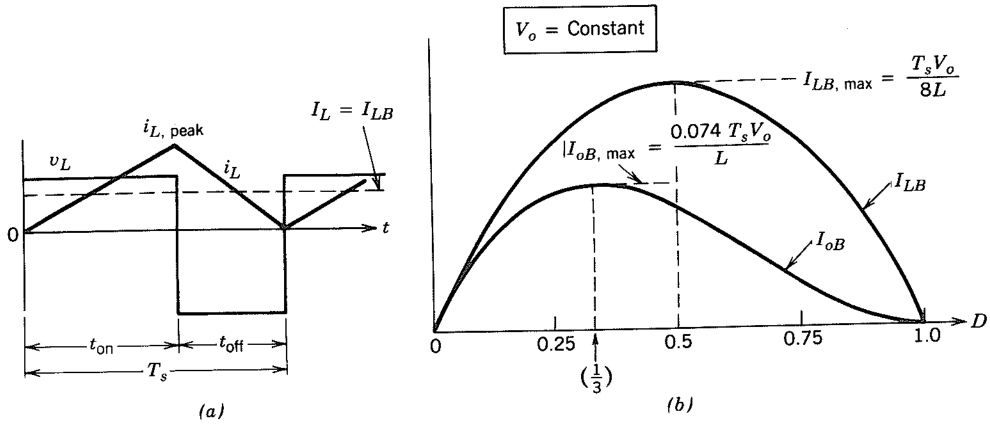

Figure 7-13 Step-up dc–dc converter at the boundary of continuous–discontinuous conduction.

Recognizing that in a step-up converter the inductor current and the input current are the same ($i_d = i_L$) and using Eq. 7-27 and 7-28, we find that the average output current at the edge of continuous conduction is

$$I_{oB} = \frac{T_s V_o}{2L} D(1 - D)^2 \tag{7-29}$$

Most applications in which a step-up converter is used require that V_o be kept constant. Therefore, with V_o constant, I_{oB} are plotted in Fig. 7-13*b* as a function of duty ratio D. Keeping V_o constant and varying the duty ratio imply that the input voltage is varying.

Figure 7-13*b* shows that I_{LB} reaches a maximum value at $D = 0.5$:

$$I_{LB,\max} = \frac{T_s V_o}{8L} \tag{7-30}$$

Also, I_{oB} has its maximum at $D = \frac{1}{3} = 0.333$:

$$I_{oB,\max} = \frac{2}{27} \frac{T_s V_o}{L} = 0.074 \frac{T_s V_o}{L} \tag{7-31}$$

In terms of their maximum values, I_{LB} and I_{oB} can be expressed as

$$I_{LB} = 4D(1 - D) I_{LB,\max} \tag{7-32}$$

and

$$I_{oB} = \frac{27}{4} D(1 - D)^2 I_{oB,\max} \tag{7-33}$$

Figure 7-13*b* shows that for a given D, with constant V_o, if the average load current drops below I_{oB} (and, hence, the average inductor current below I_{LB}), the current conduction will become discontinuous.

7-4-3 DISCONTINUOUS-CONDUCTION MODE

To understand the discontinuous-current-conduction mode, we would assume that as the output load power decreases, V_d and D remain constant (even though, in practice, D

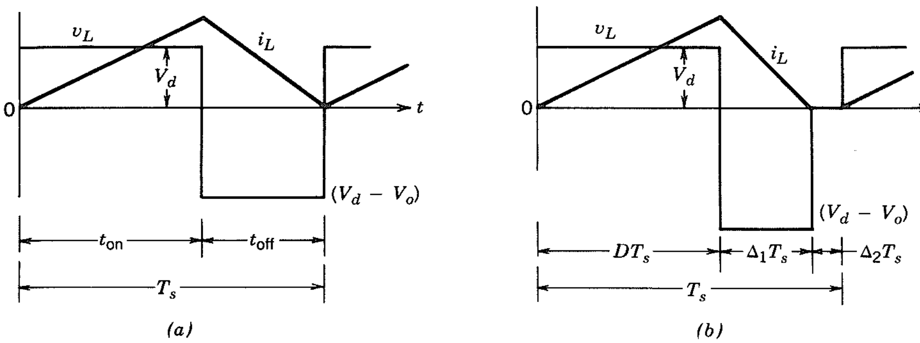

Figure 7-14 Step-up converter waveforms: (*a*) at the boundary of continuous–discontinuous conduction; (*b*) at discontinuous conduction.

would vary in order to keep V_o constant). Figure 7-14 compares the waveforms at the boundary of continuous conduction and discontinuous conduction, assuming that V_d and D are constant.

In Fig. 7-14*b*, the discontinuous current conduction occurs due to decreased P_o $(=P_d)$ and, hence, a lower I_L $(=I_d)$, since V_d is constant. Since $i_{L,\text{peak}}$ is the same in both modes in Fig. 7-14, a lower value of I_L (and, hence a discontinuous i_L) is possible only if V_o goes up in Fig. 7-14*b*.

If we equate the integral of the inductor voltage over one time period to zero,

$$V_d D T_s + (V_d - V_o)\Delta_1 T_s = 0$$

$$\therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1} \tag{7-34}$$

and

$$\frac{I_o}{I_d} = \frac{\Delta_1}{\Delta_1 + D} \quad (\text{since } P_d = P_o) \tag{7-35}$$

From Fig. 7-14*b*, the average input current, which is also equal to the inductor current, is

$$I_d = \frac{V_d}{2L} D T_s (D + \Delta_1) \tag{7-36}$$

Using Eq. 7-35 in the foregoing equation yields

$$I_o = \left(\frac{T_s V_d}{2L}\right) D \Delta_1 \tag{7-37}$$

In practice, since V_o is held constant and D varies in response to the variation in V_d, it is more useful to obtain the required duty ratio D as a function of load current for various values of V_o/V_d. By using Eqs. 7-34, 7-37, and 7-31, we determine that

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oB,\max}}\right]^{1/2} \tag{7-38}$$

In Fig. 7-15, D is plotted as a function of $I_o/I_{oB,\max}$ for various values of V_d/V_o. The boundary between continuous and discontinuous conduction is shown by the dashed curve.

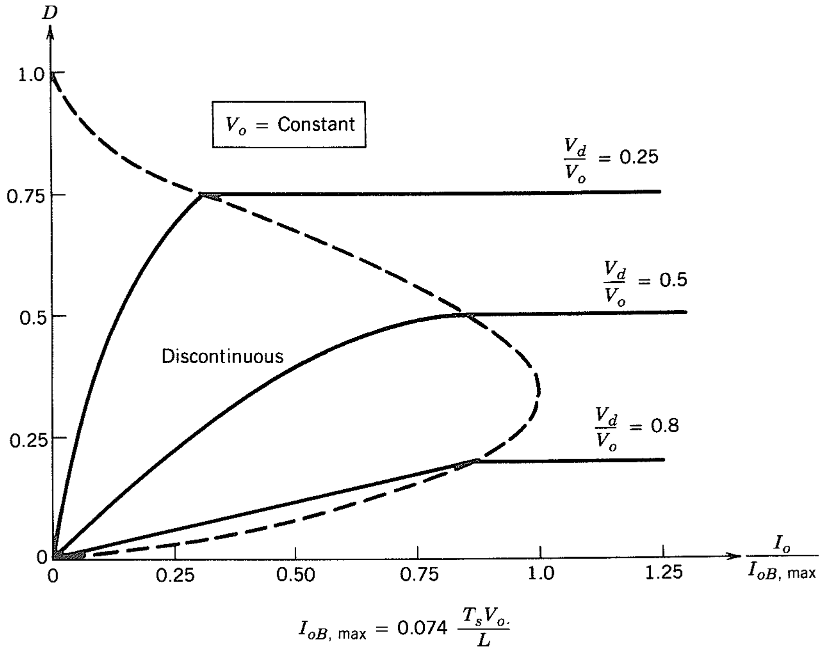

Figure 7-15 Step-up converter characteristics keeping V_o constant.

In the discontinuous mode, if V_o is not controlled during each switching time period, at least

$$\frac{L}{2} i_{L,\text{peak}}^2 = \frac{(V_d D T_s)^2}{2L} \quad \text{W-s}$$

are transferred from the input to the output capacitor and to the load. If the load is not able to absorb this energy, the capacitor voltage V_o would increase until an energy balance is established. If the load becomes very light, the increase in V_o may cause a capacitor breakdown or a dangerously high voltage to occur.

■ ***Example 7-1*** In a step-up converter, the duty ratio is adjusted to regulate the output voltage V_o at 48 V. The input voltage varies in a wide range from 12 to 36 V. The maximum power output is 120 W. For stability reasons, it is required that the converter always operate in a discontinuous-current-conduction mode. The switching frequency is 50 kHz.

Assuming ideal components and C as very large, calculate the maximum value of L that can be used.

Solution In this converter, $V_o = 48$ V, $T_s = 20$ μs, and $I_{o,\max} = 120$ W/48 V = 2.5 A. To find the maximum value of L that keeps the current conduction discontinuous, we will assume that at the extreme operating condition, the inductor current is at the edge of continuous conduction.

For the given range of V_d (12–36 V), D is in a range of 0.75–0.25 (corresponding to the current conduction bordering on being continuous). For this range of D, from Fig. 7-13*b*, I_{oB} has the smallest value at $D = 0.75$.

Therefore, by substituting $D = 0.75$ in Eq. 7-29 for I_{oB} and equating it to $I_{o,\max}$ of 2.5 A, we can calculate

$$L = \frac{20 \times 10^{-6} \times 48}{2 \times 2.5} 0.75(1 - 0.75)^2$$
$$= 9\ \mu H$$

Therefore, if $L = 9\ \mu\text{H}$ is used, the converter operation will be at the edge of continuous conduction with $V_d = 12$ V and $P_o = 120$ W. Otherwise, the conduction will be discontinuous. To further ensure a discontinuous-conduction mode, a smaller than 9 μH inductance may be used. ■

7-4-4 EFFECT OF PARASITIC ELEMENTS

The parasitic elements in a step-up converter are due to the losses associated with the inductor, the capacitor, the switch, and the diode. Figure 7-16 qualitatively shows the effect of these parasitics on the voltage transfer ratio. Unlike the ideal characteristic, in practice, V_o/V_d declines as the duty ratio approaches unity. Because of very poor switch utilization at high values of duty ratio (as discussed in Section 7-8), the curves in this range are shown as dashed. These parasitic elements have been ignored in the simplified analysis presented here; however, these can be incorporated into circuit simulation programs on computers for designing such converters.

7-4-5 OUTPUT VOLTAGE RIPPLE

The peak-to-peak ripple in the output voltage can be calculated by considering the waveforms shown in Fig. 7-17 for a continuous mode of operation. Assuming that all the ripple current component of the diode current i_D flows through the capacitor and its average value flows through the load resistor, the shaded area in Fig. 7-17 represents charge ΔQ. Therefore, the peak–peak voltage ripple is given by

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} \quad \text{(assuming a constant output current)}$$
$$= \frac{V_o}{R}\frac{DT_s}{C} \tag{7-39}$$

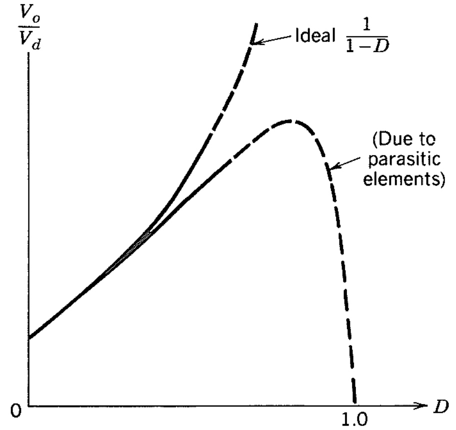

Figure 7-16 Effect of parasitic elements on voltage conversion ratio (step-up converter).